

ON INTEGRABILITY OF DYNAMIC EQUATIONS OF SPATIAL PENDULUM MOTION IN A NONCONSERVATIVE FORCE FIELD

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ABSTRACT

We systematize some results on the study of the equations of spatial motion of dynamically symmetric fixed rigid bodies-pendulums located in a nonconservative force fields. The form of these equations is taken from the dynamics of real fixed rigid bodies placed in a homogeneous flow of a medium. In parallel, we study the problem of a spatial motion of a free rigid body also located in a similar force fields. Herewith, this free rigid body is influenced by a nonconservative tracing force; under action of this force, either the magnitude of the velocity of some characteristic point of the body remains constant, which means that the system possesses a nonintegrable servo constraint, or the center of mass of the body moves rectilinearly and uniformly; this means that there exists a nonconservative couple of forces in the system.

Earlier, the author already proved the complete integrability of the equations of a plane-parallel motion of a fixed rigid body--pendulum in a homogeneous flow of a medium under the jet flow conditions when the system of dynamical equations possesses a first integral, which is a transcendental (in the sense of the theory of functions of a complex variable, i.e., it has essential singularities) function of quasi-velocities. It was assumed that the interaction of the medium with the body is concentrated on a part of the surface of the body that has the form of a (one-dimensional) plate.

In sequel, the planar problem was generalized to the spatial (three-dimensional) case, where the system of dynamical equations has a complete set of transcendental first integrals. It was assumed that the interaction of the homogeneous medium flow with the fixed body (the spherical pendulum) is concentrated on a part of the body surface that has the form of a planar (two-dimensional) disk.

In this activity, the results relate to the case where all interaction of the homogeneous flow of a medium with the fixed body is concentrated on that part of the surface of the body, which has the form of a two-dimensional disk, and the action of the force is concentrated in a direction perpendicular to this disk. These results are systematized and are presented in invariant form. We prove that our system possesses the full set of transcendental first integrals expressing through the finite combination of elementary functions.

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